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A Timoshenko beam model for cantilevered piezoelectric energy harvesters

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Abstract
Piezoelectric bimorph cantilevered beams are often used as energy harvesting devices. These devices are desired for, among other applications, remote sensing and animal tracking due to their potential for eliminating the need for battery replacement. Existing models of piezoelectric bimorph cantilevered beams have proved to describe the dynamics of slender beams at high frequencies accurately. In this paper, a Timoshenko model of transverse piezoelectric beam vibration is developed to address these limitations. Exact expressions for the voltage, current, power, and tip deflection of the piezoelectric beam are derived. Subsequently, several case studies are presented that examine the frequency response of vibration-based energy harvesters using this model. It is shown that the predicted responses converge towards previously derived Euler–Bernoulli beam models under certain limiting conditions. The Timoshenko model shows that the Euler–Bernoulli model severely over-predicts the tip displacement and consequently the power transduction of a cantilevered piezoelectric bimorph at low length-to-width aspect ratios.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The recent increase in the demand for wireless devices coupled with the decrease in their power requirements has precipitated an explosion in research and development of practical means to provide sufficient energy to meet their application needs. Due to their relatively low energy densities, on-board batteries may be impractical for certain small-scale applications (Warneke *et al* 2001, Reissman *et al* 2007). For distributed sensor networks with a large number of nodes, batteries may be infeasible due to the cost of replacement. A means of making these systems entirely self-sufficient through energy replenishment from their local surroundings may be the key to the proliferation of wireless technology into new environments.

Vibration-based energy harvesting—the conversion of ambient vibrations into useful electrical energy—has received significant attention due to the ubiquity of untapped vibrational energy available in or around most manmade systems (Roundy *et al* 2003). The attention drawn towards this under-utilized energy source has spurred research on several methods of electromechanical transduction (Beeby *et al* 2006), including electromagnetic induction (e.g. Glynne-Jones *et al* 2004), electrostatic varactance (e.g. Mitcheson *et al* 2004), and the piezoelectric effect (e.g. Anton and Sodano 2007), the latter being the focus of this study. Although the power harvested is generally small compared to the power required to operate sensors and RF communications continuously, several researchers have demonstrated that, through careful energy budgeting, piezoelectric energy harvesting can provide a viable design solution for maintenance-free, wireless electronics. For example, Kymissis *et al* (1998) have investigated energy harvesting from walking and designed power management circuitry for intermittent RFID transmission. Self-powered, wireless sensors have also been produced for temperature and humidity measurement (Arms *et al* 2005) and machinery acceleration monitoring (Discenzo *et al* 2006).

The typical piezoelectric energy harvester geometry consists of a cantilevered beam with one or two layers

